Interest Rates and Duration

We will discuss the following

- Zero rates
- Par yields
- Forward rates
- Day count convention
- How prices of bonds and treasury bills are quoted
- Duration measure
- Interest rate future markets
- Treasure bond futures etc.

Types of Rates

Note: Higher the credit risk, implies higher interest rate

- Treasury rates: Rate of interest applicable to borrowing by a government in its own currency. Regarded as risk free interest rate
- LIBOR rates: London Interbank Offer Rate, general higher than treasury rates. Similarly in India we have the MIBOR (Mumbai Interbank Offer Rate), which can be obtained from any financial paper or financial home page link
- Repo rates, i.e. Repurchase agreement. Higher than Treasury rates. Examples overnight repo, term repo

MIBID and MIBOR

As on 7th August 2006

Category	Time	MIBID	SD(MIBID)	MIBOR	SD(MIBOR)
Overnight	0940	6.00	0.0029	6.10	0.0001
14 day	1130	6.00	0.0833	6.36	0.0054
1 Month	1130	6.25	0.0544	6.63	0.0409
3 Month	1130	6.58	0.0781	7.16	0.0576

Where
MIBID: Mumbai Interbank Bid Rate
MIBOR: Mumbai Interbank Offered Rate

Interest Rates & Duration

- n-year zero rate/n-year zero/n-year spot rate: Interest rate earned on investment that starts today and ends n years from hence.
- Note

a) There is ***NO*** intermediate payments

b) Rates on government bonds are not exactly equal to this rate as the government bonds have intermediate payments in form of coupons

Bond Pricing and Bond Yield

- How do you calculate the price of a bond, if time to maturity is T years and it pays semiannual or quarterly or monthly coupons? Remember the zero rate for different maturities used to calculate the value of coupons at different times are not equal (Refer the different MIBID and MIBOR rates).
- The yield on a coupon bearing bond is the discount rate that equates the cash flows on the bond to its market value.

Bond pricing example

Maturity (years)	Zero Rate (% cont comp)
0.5	5.0
1.0	5.8
1.5	6.4
2.0	6.8

Bond Pricing

Consider the example where we have a two year (i.e., maturing in two years) treasury bond with a principal of $100 and it provides a coupon at the rate of 6% per annum with coupons being paid semi-annually. We are to find the **price of the bond** as of today

Bond Pricing

To calculate the **cash price** of a bond we discount each cash flow at the appropriate zero rate

$$\left(\frac{6}{2}\right)e^{-0.05\times 0.5}+\left(\frac{6}{2}\right)e^{-0.058\times 1.0}+\left(\frac{6}{2}\right)e^{-0.064\times 1.5}$$

$$+\left(100+\frac{6}{2}\right)e^{-0.068\times 2.0}=98.39$$

Bond Yield

The bond yield is the discount rate that makes the present value of the cash flows on the bond equal to the market price of the bond. Suppose that the market price of the bond in our example equals its theoretical price of 98.39. Then the bond yield is given by solving

$$\left(\frac{6}{2}\right)e^{-y\times 0.5}+\left(\frac{6}{2}\right)e^{-y\times 1.0}+\left(\frac{6}{2}\right)e^{-y\times 1.5}$$
$$+\left(100+\frac{6}{2}\right)e^{-y\times 2.0}=98.39$$

to get y=0.0676 or 6.76%.

Bond Yield

- Use Newton-Raphson method or Runga-Kutta method etc.
- Newton Raphson method
  a) Start with any value x_0 for solving f(x)=0.
  b) Use successively $x_{i+1}=x_i-f(x_i)/f'(x_i)$ to find x_1, x_2, x_3,. …. and so on.

Par Yield

- The par yield for a certain maturity is the coupon rate that causes the bond price to equal its face value.
- In our example we solve

$$\frac{c}{2}e^{-0.05\times 0.5}+\frac{c}{2}e^{-0.058\times 1.0}+\frac{c}{2}e^{-0.064\times 1.5}$$

$$+\left(100+\frac{c}{2}\right)e^{-0.068\times 2.0}=100$$

$$\text{to get } c=6.87 \text{ (with s.a. compounding)}$$

Par Yield

In general if

- m is the number of coupon payments per year
- d is the present value of $1 received at maturity
- A is the present value of an annuity of $1 on each coupon date

$$100 = A * \left(\frac{c}{m} \right) + 100 * d$$

Continuous Compounding vis-à-vis Other Rates of Compounding

If

- r_c is the continuous compounding rate.
- r_m is the equivalent rate with compounding m times per annum.
- n is the number of years.
- m is the number of times of compounding for r_m.

$$r_c = m \log_e \left(1 + \frac{r_m}{m} \right)$$

Determining the Zero Curve

Bond Principal (dollars)	Time to Maturity (years)	Annual Coupon (dollars)	Bond Price (dollars)
100	0.25	0	97.5
100	0.50	0	94.9
100	1.00	0	90.0
100	1.50	8	96.0
100	2.00	12	101.6

Determining the Zero Curve

- We assume that ½ of the stated coupon is assumed to be paid every six months

The Bootstrapping the Zero Curve

- An amount (100.0-97.5)=2.5 can be earned on 97.5 during 3 months.
- The 3-month rate is 4 times (2.5/97.5) or 10.256% with quarterly compounding
- This is 10.127% with continuous compounding

The Bootstrapping the Zero Curve

- An amount (100.0-94.9)=5.1 can be earned on 94.9 during 6 months.
- The 6-month rate is 2 times (5.1/94.9) or 10.748% with semi annual compounding
- This is 10.469% with continuous compounding

The Bootstrapping the Zero Curve

- An amount (100.0-90.0)=10.0 can be earned on 90.0 during 12 months.
- The 12-month rate is 1 times (10.0/90.0) or 11.111% with annual compounding
- This is 10.536% with continuous compounding

The Bootstrap Method

To calculate the 1.5 year rate we solve

$$\left(\frac{8}{2}\right)e^{-0.10469\times 0.5}+\left(\frac{8}{2}\right)e^{-0.10536\times 1.0}$$
$$+\left(100+\frac{8}{2}\right)e^{-R\times 1.5}=96$$

and R = 0.10681 or 10.681%

The Bootstrap Method

To calculate the 2.0 year rate we solve

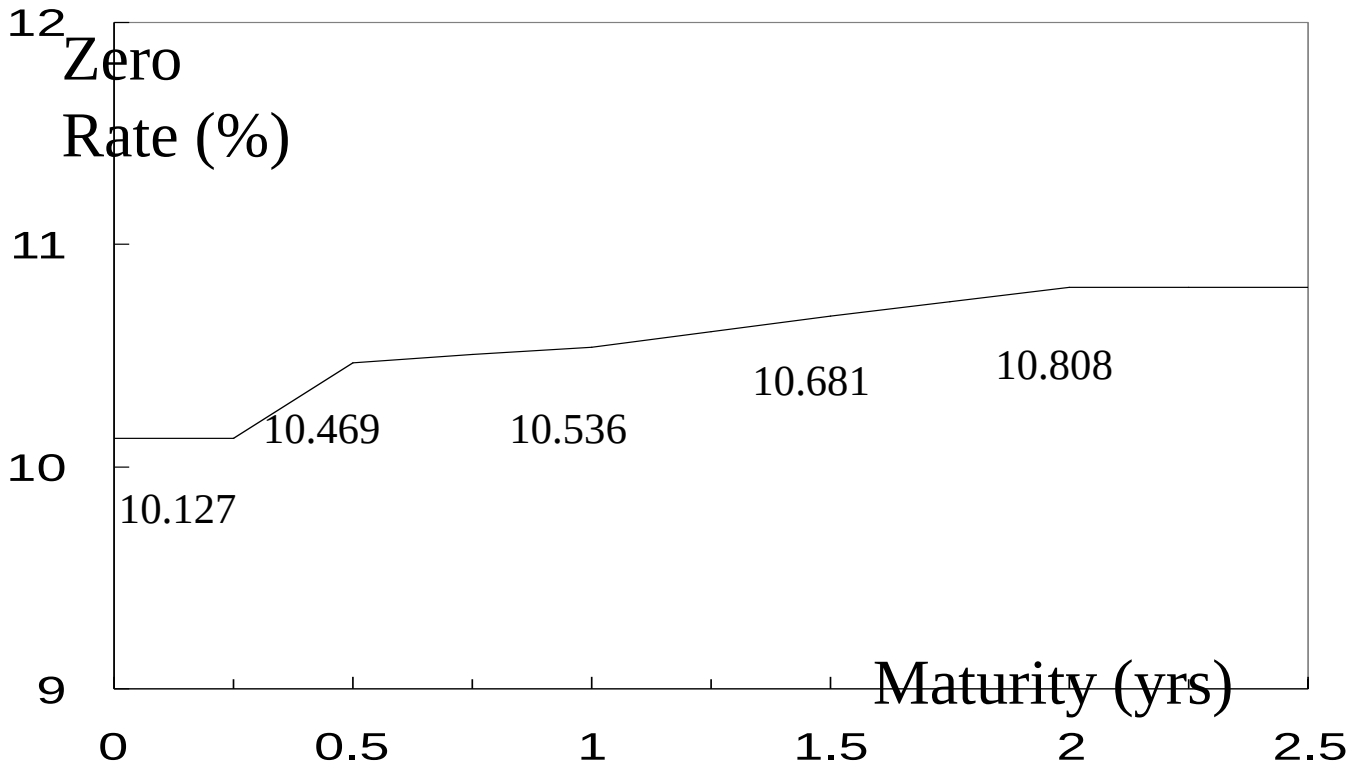
$$\left(\frac{12}{2}\right)e^{-0.10469\times 0.5}+\left(\frac{12}{2}\right)e^{-0.10536\times 1.0}$$
$$+\left(\frac{12}{2}\right)e^{-0.10681\times 1.5}+\left(100+\frac{12}{2}\right)e^{-R\times 2.0}=101.6$$

and R = 0.10808 or 10.808%

Continuously Compounded Zero Rates

Maturity (yrs)	Zero Rate (Continuous Compounding
0.25	10.127
0.50	10.469
1.00	10.536
1.5	10.681
2.00	10.808

Zero Curve Calculated from the Data



Forward Rates

A forward interest rate is the interest rate implied by current zero rates for a specified future time period

Calculation of Forward Rates

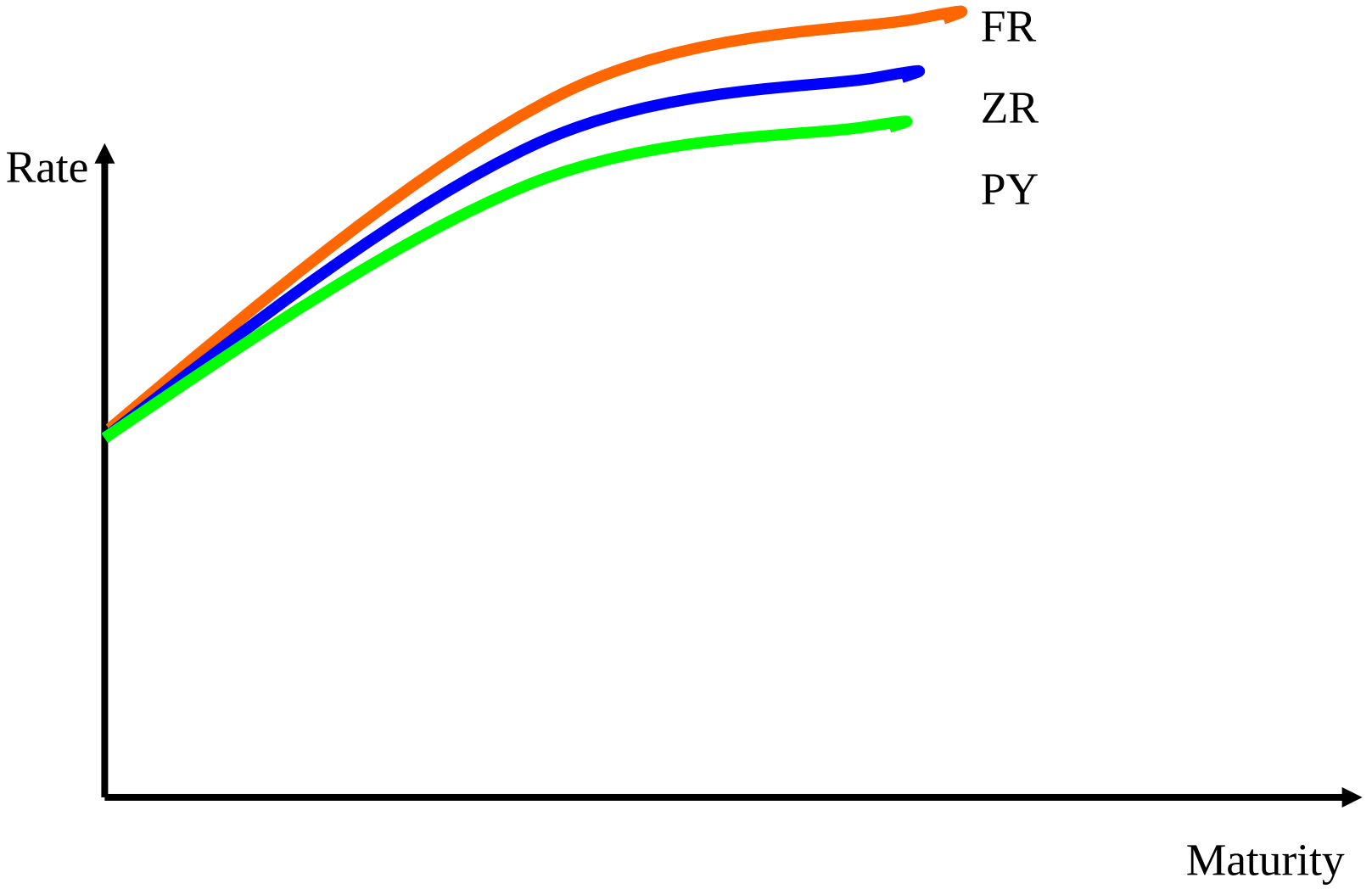
Year (n)	Zero Rate for an n-year Investment (% per annum)	Forward Rate for nth Year (% per annum)
1	10.0	
2	10.5	11.0
3	10.8	11.4
4	11.0	11.6
5	11.1	11.5

Formula for Forward Rates

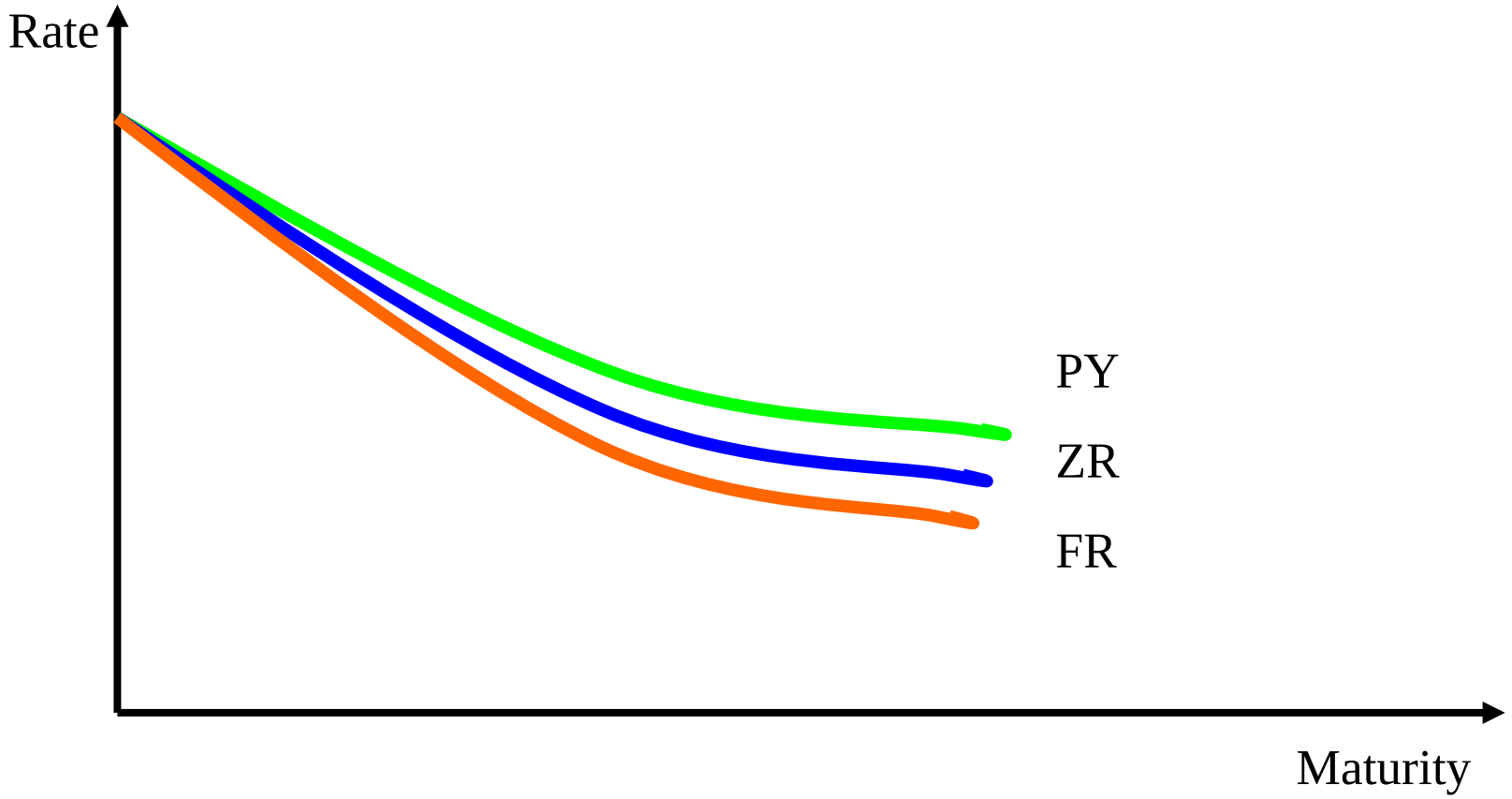
- Suppose that the ***zero rates*** for maturities T_1 and T_2 are R_1 and R_2 respectively, with ***both rates continuously compounded***.
- The forward rate for the period between times T_1 and T_2 is

$$\frac{R_2 T_2 - R_1 T_1}{T_2 - T_1}$$

Sloping of the different rates curves, when Yield curve upward sloping



Sloping of the different rates curves when Yield curve is downward sloping



Instantaneous Forward Rate

The instantaneous forward rate for a maturity T is the forward rate that applies for a very short time period starting at T. It is

$$R + T\frac{\partial R}{\partial T}$$

where R is the T-year rate

Forward Rate Agreement

A ***forward rate agreement*** (FRA) is a forward contract where the parties agree that a certain interest rate will apply to a certain principal during a specified future time period

Theories of the Term Structure

What determines the shape of the zero Curve?

- ***Expectations Theory***: Long term interest rates should reflect expected future short term interest rates, i.e., forward rates equal expected future zero rates
- ***Market Segmentation Theory***: Short, medium and long rates determined independently of each other
- ***Liquidity Preference Theory***: Forward rates should always be higher than expected future zero rates

Some other definitions

- Day count conventions
- Quotations
  a) Bonds
  b) Treasury Bills

Duration

Duration of a bond is a measure of how long on average the holder of he bond has to wait before receiving cash payments. Hence a zero coupon bond that matures in “n” years has a duration of “n” years. However we can say without any problem that a coupon bearing bond maturing in “n” years has a duration ***LESS*** than “n”

Duration

Duration of a bond that provides cash flow c_i at time t_i is

$$\sum_{i=1}^{n} t_i \left[\frac{c_i e^{-yt_i}}{B} \right]$$

where B is its price and y is its yield (continuously compounded). Note the term in the bracket is the ratio of the present value of the payment at time t_i to the bond price.

This leads to

$$\frac{\delta B}{B} = -D\delta y$$

Example 4.1

Suppose that zero interest rates with continuous compounding are as follows

Maturity (year)	Rate (% p.a)
1	12.0
2	13.0
3	13.7
4	14.2
5	14.5

Calculate the forward interest rates for the 2nd, 3rd, 4th and 5th years

Example 4.2

A five year bond with a yield of 11% (continuously compounding) pays an 8% coupon at the end of each year. Then find the following: (i) bond price, (ii) bond duration, (iii) using this duration calculate the effect on the bond′s price for a 0.2% decrease in its yield and (iv) recalculate the bond′s price on the basis of a 10.8% per annum yield and verify the result is in agreement with your answer found out in (iii).

Chapter 5: Swaps

We will cover and discuss about

- Interest rate swaps
- Currency rate swaps
- Valuation of currency rate swaps

Swaps

Agreement between two companies to exchange cash flows at specified future times according to certain specified rules.

Example: A forward contract is a simple example of a swap.

Swaps

Whereas a forward contract leads to exchange of cash flows on just one future date, swaps typically lead to cash flow exchanges on several future dates.

Swaps

Example: On 1st March 1999 a company enters into long forward contract to buy 100 ounces of gold for $300 per ounce. Immediately on getting the gold it can short the gold for $100*S_T$. Note it leads to a cash flow on just ONE future date. But SWAPS typically lead to cash flows on several future dates.

Swaps

Types

- Interest Swaps (Simple one called "***Plain Vanilla***"): The company basically agrees to pay cash flows equal to an interest amount which is calculated at a predetermined fixed interest rate on a notional principal amount for a number of years. In return, it receives an interest amount which is calculated at a floating interest rate on the same notional principal amount for the same period of time
- Currency Swaps

Swaps

Important: For this portion, when ever we quote any interest rates they are % per annum and payable annually and nothing to do with continuous compounding case

Swaps

Example: Consider a hypothetical 3 year swap initiated on March, 5th 2001 between MS and Intel. We suppose MS agrees to pay Intel an interest rate of 5% per annum on a notional principal amount of $100 million, and in return Intel agrees to pay MS the 6-month LIBOR rate on the same notional principal. We assume that the agreement specifies that payments are to be exchanged every 6 months

Swaps
Cash flow for MS

		---------Millions of Dollars---------		
	LIBOR	*FLOATING*	*FIXED*	Net
Date	Rate	Cash Flow	Cash Flow	Cash Flow
Mar.5, 2001	4.2%			
Sept. 5, 2001	4.8%	+2.10	–2.50	–0.40
Mar.5, 2002	5.3%	+2.40	–2.50	–0.10
Sept. 5, 2002	5.5%	+2.65	–2.50	+0.15
Mar.5, 2003	5.6%	+2.75	–2.50	+0.25
Sept. 5, 2003	5.9%	+2.80	–2.50	+0.30
Mar.5, 2004		+2.95	–2.50	+0.45

Swaps

For this example if we see very carefully we notice that the cash flows in the 3rd column are basically the cash flows for a long position in a floating rate bond. The cash flows in the 4th column are the cash flows from a short position in a fixed rate bond. The table can be regarded as the exchange of a fixed rate bond for a floating rate bond. **MS is long a floating rate and short a fixed rate bond. While for Intel it is just the opposite**

Typical Uses of an Interest Rate Swap

- Converting a liability from
  - fixed rate to floating rate
  - floating rate to fixed rate
- Converting an investment from
  - fixed rate to floating rate
  - floating rate to fixed rate

Swaps

Use a swap to transform a **liability**

Suppose MS has arranged to borrow $100 million at LIBOR plus 10 basis points, so interest rate is LIBOR+0.1%. After MS has gone into the swap, it has the following **three** sets of cash flows

- It pays LIBOR+0.1% to its outside lender
- It receives LIBOR under the term of the contract (which it has signed with Intel)
- It pays 5% under the term of the contract (which it has signed with Intel)

Swaps

Use a swap to transform a **liability**

Suppose Intel has arranged to borrow $100 million at 5.2%. After Intel has gone into the swap, it has the following **three** sets of cash flows

- It pays 5.2% to its outside lender
- It pays LIBOR under the term of the contract (which it has signed with MS)
- It receives 5% under the term of the contract (which it has signed with MS)